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EFFECTIVENESS-ROBUSTNESS COMPROMISE FOR OPTIMUM DESIGN OF MTMD SYSTEMS THROUGH MULTI OBJECTIVE GA OPTIMIZATION

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ABSTRACT

Multiple tuned mass dampers (MTMD) consisting of many TMDs with equally distributed natural frequencies are considered for attenuating effectively the response of uncertain structures to harmonic force excitation.

Considering the uncertainty in the natural frequency of the structural mode being controlled, the concept of nominal effectiveness is not the only aspect that characterizes performance; the robust effectiveness must also be addressed. In this investigation, effectiveness and robustness are considered as the two objectives of optimization in the design of MTMD systems. It is shown that by appropriately indexing these two (usually) conflicting criteria the optimum design of MTMD systems can be formulated as a two-objective optimization problem.

In this paper a methodology is proposed to efficiently formulate the compromise-decision making process of MTMD system design. This methodology includes appropriate indexing of MTMD effectiveness and robustness concepts, formulating respective objective (fitness) functions and obtaining the resulting non-dominated set of solutions. This set of solutions can be conveniently found through GA implementation and presented as Pareto fronts to help designers find an effective MTMD, while maintaining acceptable robustness.

1. INTRODUCTION

The tuned mass damper (TMD) is one of the most basic passive control devices for vibration suppression in engineering structures. After the introduction and suggestion of TMD optimized utilization by pioneers in the area, e.g. Brock [1] and Den Hartog [2], attention was directed to broaden this area. Soon, it was recognized that there are some disadvantages with too sensitive a single optimized TMD. To overcome this disadvantage of high sensitivity to offset in the optimal tuning of a STMD, dual tuned mass dampers [3] and multiple tuned mass dampers [4] were proposed. Now, this well-matured field is not only widely accepted in theoretical research but is frequently applied and implemented in full-scale engineering structures.

In the present investigation, we associate the concept of MTMD systems performance with two major criteria, namely effectiveness and robustness to the uncertainty in the natural frequency of the structure. Toward the purpose of optimizing MTMD systems, there are two points that must be considered about these two criteria:

- First, a quantitative index must be assigned to each criterion which precisely reflects the underlying qualitative concept.
- Then, the optimization of MTMD performance must be carefully formulated with regard to the relative behavior of the optimization criteria in the design space.

The effectiveness, as the nominal performance, can be most generally indexed by the H_2 and/or H_∞ norms of the transfer function of the structure-MTMD dynamic system from disturbance input to the regulated output [5]. For the case of a SISO system (a SDOF-MTMD system with the excitation input and structure displacement or acceleration response as output), the worst case index of performance (H_∞) is simply the peak frequency response. This concept is commonly referred to as the maximum dynamic magnification factor (max.DMF) of the structure-MTMD system in the literature, with the objective being its minimization (min.max.DMF).

On the other hand, robustness, although well-identified as a performance criterion, has merely received qualitative comments from most researchers. Yamaguchi and Harnpornchai [6] associated the concept of robustness with the qualitative behavior of the max.DMF to changes in the natural frequency error; Kareem and Kline [7] interpreted this concept in a similar fashion; Li [8] pointed out the flatness of response curves as the characteristic of more robust MTMD systems; and Abe and Fujino [9], in an attempt to go slightly beyond qualitative description, proposed the reserve bandwidth criterion. However, the most suited, available criterion for quantitative evaluation (and objective function formulation) is the robustness index $\Re$ which was proposed by the authors of the present paper in [10].

In the aforementioned investigation by the authors [10], it was shown that effectiveness and robustness have conflict and hence can not be properly combined into a single objective. In the present paper, considering this relative conflicting behavior of these two optimization criteria with variation of MTMD design parameters, it is proposed to formulate the optimum design of MTMD systems as a two-objective optimization problem. It will be also shown that the set of solutions to this problem can be conveniently presented through the concept of Pareto optimality [11]. A genetic algorithm has been implemented due to its appealing ability to converge to the Pareto optimal set rather than a single Pareto optimal point.

2. STRUCTURE-MTMD SYSTEM

The structure is represented by its mode-generalized system in the specific vibration mode being controlled using the mode reduced-order method. The structure together with the MTMD form a (n+1)-DOF system. Although the derivation of a MTMD formulation is not the concern of this paper, choosing a specific model along with its assumptions and formulation facilitates the illustration of the concepts regarding the objectives of this investigation. Figure 1 shows such a model:

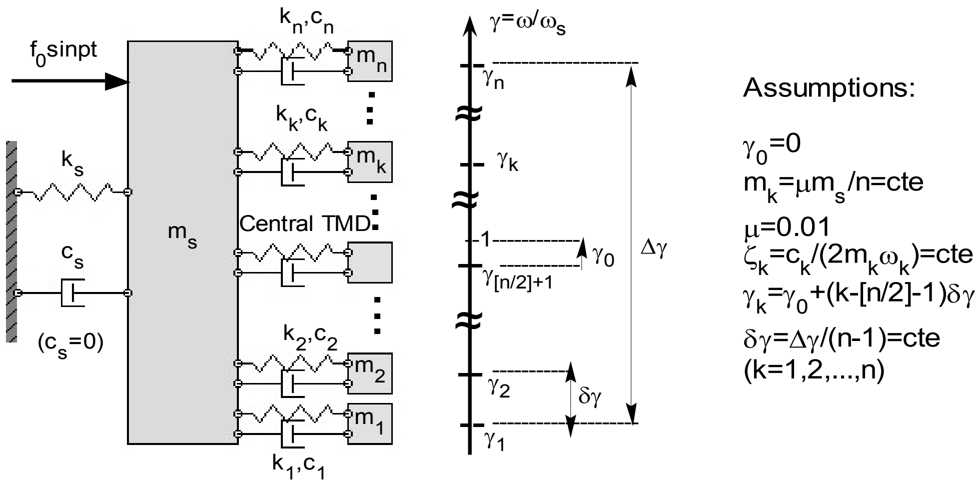

Figure 1. Structure-MTMD system model and assumptions

According to the above assumptions, for the system shown in the figure, the dynamic magnification factor (DMF) of the structural response is derived as [6]:

$$DMF = \frac{1}{\sqrt{\mathrm{Re}^2(Z) + \mathrm{Im}^2(Z)}} \tag{1}$$

where:

$$\mathrm{Re}(Z) = 1 - \left(\frac{p}{\omega_s}\right)^2 - \sum_{k=1}^{n} \frac{\mu_k \left(\frac{p}{\omega_s}\right)^2 \left[\gamma_k^{\,2}\left\{\gamma_k^{\,2} - \left(\frac{p}{\omega_s}\right)^2\right\} + \left\{2\zeta_k\gamma_k \frac{p}{\omega_s}\right\}^2\right]}{\left\{\gamma_k^{\,2} - \left(\frac{p}{\omega_s}\right)^2\right\}^2 + \left\{2\zeta_k\gamma_k \frac{p}{\omega_s}\right\}^2}$$

$$\mathrm{Im}(Z) = 2\zeta_s \frac{p}{\omega_s} + \sum_{k=1}^{n} \frac{2\mu_k\zeta_k\gamma_k \left(\frac{p}{\omega_s}\right)^2}{\left\{\gamma_k^{\,2} - \left(\frac{p}{\omega_s}\right)^2\right\}^2 + \left\{2\zeta_k\gamma_k \frac{p}{\omega_s}\right\}^2}$$

n	Total number of TMD's	ω_s	Natural freq. of the structure
$\delta\gamma$	Spacing of TMD's freq. (In this investigation: uniformly distributed)	p (or ω)	External force freq.
$\Delta\gamma$	MTMD (non-dimensionalized) freq. range	μ	Total mass ratio (set to 0.01 throughout this study)
γ_0	Offset freq. (In this investigation: set to zero)	μ_k (=μ/n)	Mass ratio of the k^{th} TMD
γ_k	Freq. ratio of the k^{th} TMD	ζ_k (=ζ)	Damping ratio of the k^{th} TMD

Table 1: Nomenclature for the Structure-MTMD system model

3. OBJECTIVES

In this section the two objectives of optimization will be specified. These two objectives are the two aforementioned criteria of performance, evaluated through appropriate indices that would be presented.

3.1 Effectiveness indexed by max.DMF

The most broadly accepted and applied objective of effectiveness optimization is the minimization of the maximum dynamic magnification factor (min.max.DMF) of the structure-MTMD system. This might involve the minimization of the worst-case displacement DMF or acceleration DMF under external disturbing force or ground excitation (min.max.DDMF; min.max.ADMF) [8]. In this investigation, the peak value of the DDMF calculated from equation (1) over frequency is the first minimization objective function utilized in the GA optimization.

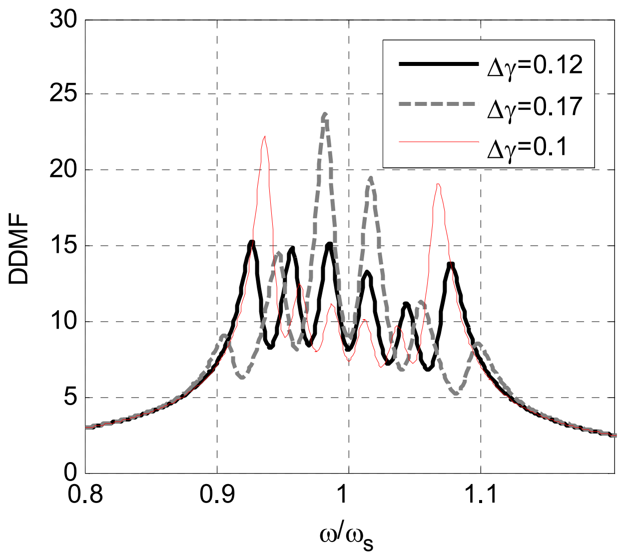

Figure 1. DDMF Response for MTMD with ζ=0.01 and n=5

It should be noted that the specific analytical formulation used for computation of DDMF causes no loss of generality in the optimization methodology, since for any other problem it is simply necessary to modify this sub-function to calculate the peak frequency response with the specific assumptions and conditions imposed on the problem.

The influence of the MTMD design parameters (frequency range, TMD damping and TMDs total number) on the effectiveness is studied according to the max.DMF index and is available in the literature (e.g. see ref [6]).

3.2 Robustness indexed by $\Re$

The flatness of the max.DMF curve versus error in natural frequency is a good qualitative index of the robustness of a MTMD-system in the presence of uncertainty in the natural frequency of the structure. To derive a quantitative index from this concept, it was proposed [10] to measure robustness through an averaging of angles defined on this curve, as shown in Figure 2. Applying the axes scaling and definition of angles shown in this figure, the average angles of robustness with respect to positive and negative (right and left) errors would be expressed as follows.

$$\begin{aligned}
\left(\alpha_{\mathrm{R}}\right)_{ave} &= \frac{1}{\mathrm{N}}\sum_{i=1}^{\mathrm{N}}\cot^{-1}\left\{\frac{1}{\mathrm{e_i}}\left[\mathrm{maxDMF_{dB}}\left(\mathrm{e_i}\right)-\mathrm{maxDMF_{dB}}\left(0\right)\right]\right\} \\
\left(\alpha_{\mathrm{L}}\right)_{ave} &= \frac{1}{\mathrm{N}}\sum_{i=1}^{\mathrm{N}}\cot^{-1}\left\{\frac{1}{\mathrm{e_i}}\left[\mathrm{maxDMF_{dB}}\left(\text{-}\mathrm{e_i}\right)-\mathrm{maxDMF_{dB}}\left(0\right)\right]\right\}
\end{aligned} \tag{2}$$

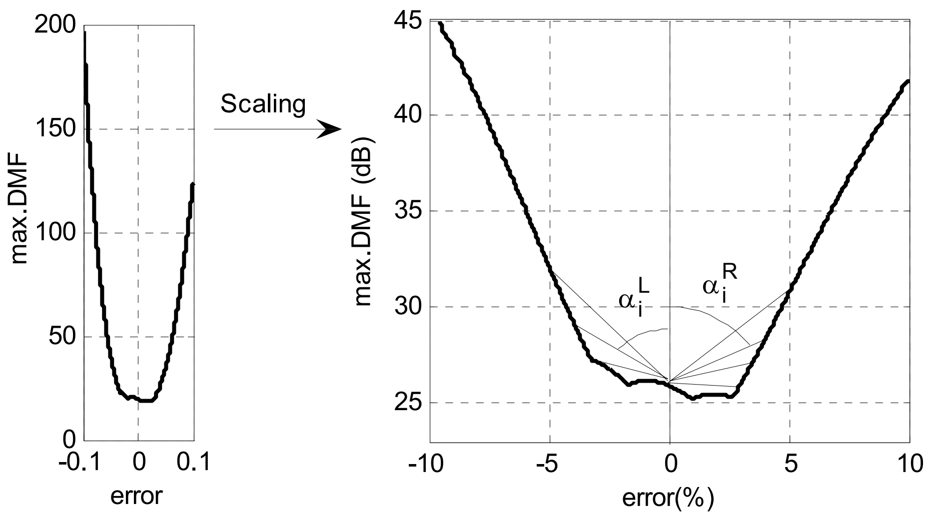

Figure 2. Scaling and definition of angles

Assuming the relative error percentage e_i to vary from zero to e_R, e.g. 5%, in both directions (with the intervals: [0,e_R] and [-e_R,0] discretized to N points), applying the dB unit conversion and also normalizing by the right angle of $\pi/2$ one can obtain the left and right robustness as follows.

$$\begin{aligned}
\Re_{\mathrm{R}} &= \frac{2}{\pi \mathrm{N}}\sum_{i=1}^{\mathrm{N}}\cot^{-1}\left[\frac{20\mathrm{N}}{\mathrm{e_R}.\mathrm{i}}.\log_{10}\left(\frac{\mathrm{maxDMF}\left(\mathrm{e_R}.\mathrm{i}/\mathrm{N}\right)}{\mathrm{maxDMF}\left(0\right)}\right)\right] \\
\Re_{\mathrm{L}} &= \frac{2}{\pi \mathrm{N}}\sum_{i=1}^{\mathrm{N}}\cot^{-1}\left[\frac{20\mathrm{N}}{\mathrm{e_R}.\mathrm{i}}.\log_{10}\left(\frac{\mathrm{maxDMF}\left(\text{-}\mathrm{e_R}.\mathrm{i}/\mathrm{N}\right)}{\mathrm{maxDMF}\left(0\right)}\right)\right]
\end{aligned} \tag{3}$$

It is reasonable to define the overall robustness $\Re$ as the minimum of $\Re_L$ and $\Re_R$. Since the output of cot^{-1} function, in equations (2) and (3), is assumed to be in the range: [0,π], the numerical values of $\Re$, $\Re_L$ and $\Re_R$ would be restricted to the range (0,2), with a numerical

value of unity representing relative insensitivity of effectiveness to error in ω_s. Numerical values less or greater than unity refer to decreasing or increasing behaviors of effectiveness against error, respectively. It is also interesting to note that since the offset frequency of the central TMD in the MTMD model used here was set to be zero, the positive error in the natural frequency of the structure makes the MTMD more effective. This leads to a greater value of $\Re_R$ compared to the value of $\Re_L$, making the latter dominant for indexing the overall robustness.

Figure 3 depicts the effectiveness behaviors of four MTMD's (with 0.01 damping, 5 numbers of TMDs and different frequency ranges) to changes in relative error in ω_s. As can be concluded, the numerical value of the proposed index is in perfect harmony with the qualitative judgment on robustness.

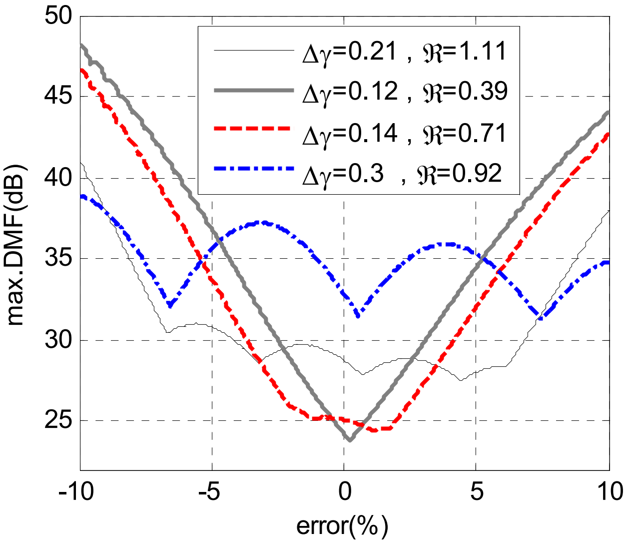

Figure 3. max.DMF vs. nat. freq. error with e_R=5% for a 5TMD (ζ=0.01)

The effect of MTMD design parameters on robustness according to this index is investigated and compared to their respective influence on effectiveness in ref. [10].

4. OPTIMIZATION

As mentioned previously, maximization of effectiveness and robustness are conflicting objectives. Hence choosing MTMD design parameters, i.e. $\Delta\gamma$, ζ and n, to simultaneously achieve both objectives is impossible, in other words, a compromise is required. This compromise can be conveniently chosen from the entire set of non-dominated solutions based on the designer's preferences.

4.1 GA Implementation

GA is a computational representation of natural selection, inspired by the concept of higher survival chance of the fittest individual in its environment through generations to find the optimal design among the others. Detailed treatment of GA methods, concepts and operators can be found in the literature [12] as it has proved its efficiency in handling various problems, including optimization of passive and active structural control, e.g. [13]. The main advantages of genetic algorithms can be summarized in that they do not require and do not depend on gradient information; they use a population of design points and randomly utilize information from each generation to the subsequent one; and, there is a potential to make the population converge to the Pareto optimal set. The latter property is a crucial appeal of GA in multi-objective optimization problems that enables us to use GA in combination with a Pareto set filter to obtain a near approximation of the entire set of non-dominated solutions.

In this investigation a two branch tournament algorithm, based on this feature of GA, is implemented for the specific problem in hand. For detailed treatment and concepts concerning general use of two branch tournament GA in multi-objective optimization, ref. [14] might be consulted. Reference [13] involves a similar application of this algorithm to an active structural control problem. The specifications of the GA implementation in the present study are listed in Table 2.

Algorithm Type		Two branch tournament
Minimization Objectives		max.DMF & $1/\Re$
Population Size		1000
Number of Generations		50
Cross-over	**Type**	Two-point
	Prob.	0.7
Mutation Probability		0.02

Table 2: GA Specifications

4.2 Results

In this investigation, the design space for the optimization problem is ζ- $\Delta\gamma$ and the criterion space is $1/\Re$-max.DMF. The results of the GA two-objective optimization are presented in these two spaces for different total numbers of TMDs in Figures 4 and 5.

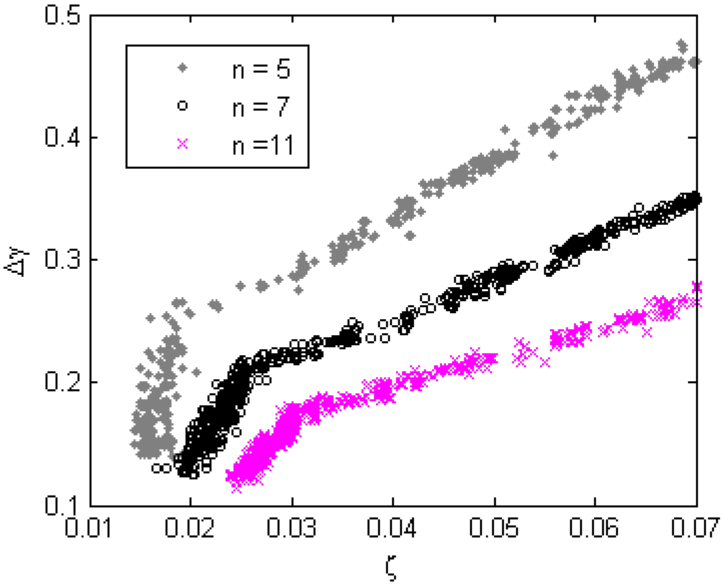

Figure 4. Pareto sets of points in the design space

Figure 5 shows also that by increasing the total number of TMDs it is possible to achieve higher robustness while maintaining a constant effectiveness, and vice versa.

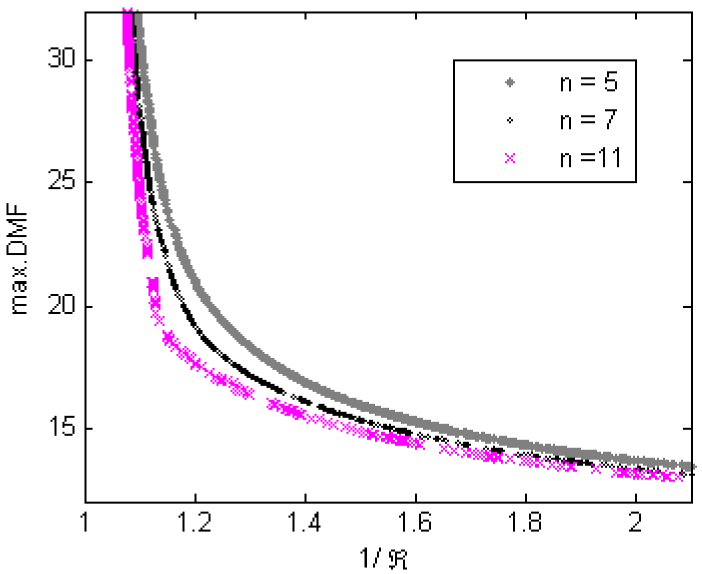

Figure 5. Pareto sets of solutions in the criterion space

In ref [10], the authors proposed a systematic methodology to select the MTMD frequency range for any fixed set of values for the TMD damping and total number. This methodology confined the selection of the frequency range between two bounds of maximum effectiveness and maximum robustness. Now we can add the Pareto set of designs to the design diagrams in order to more accurately guide the designer's choice to the non-dominated solutions.

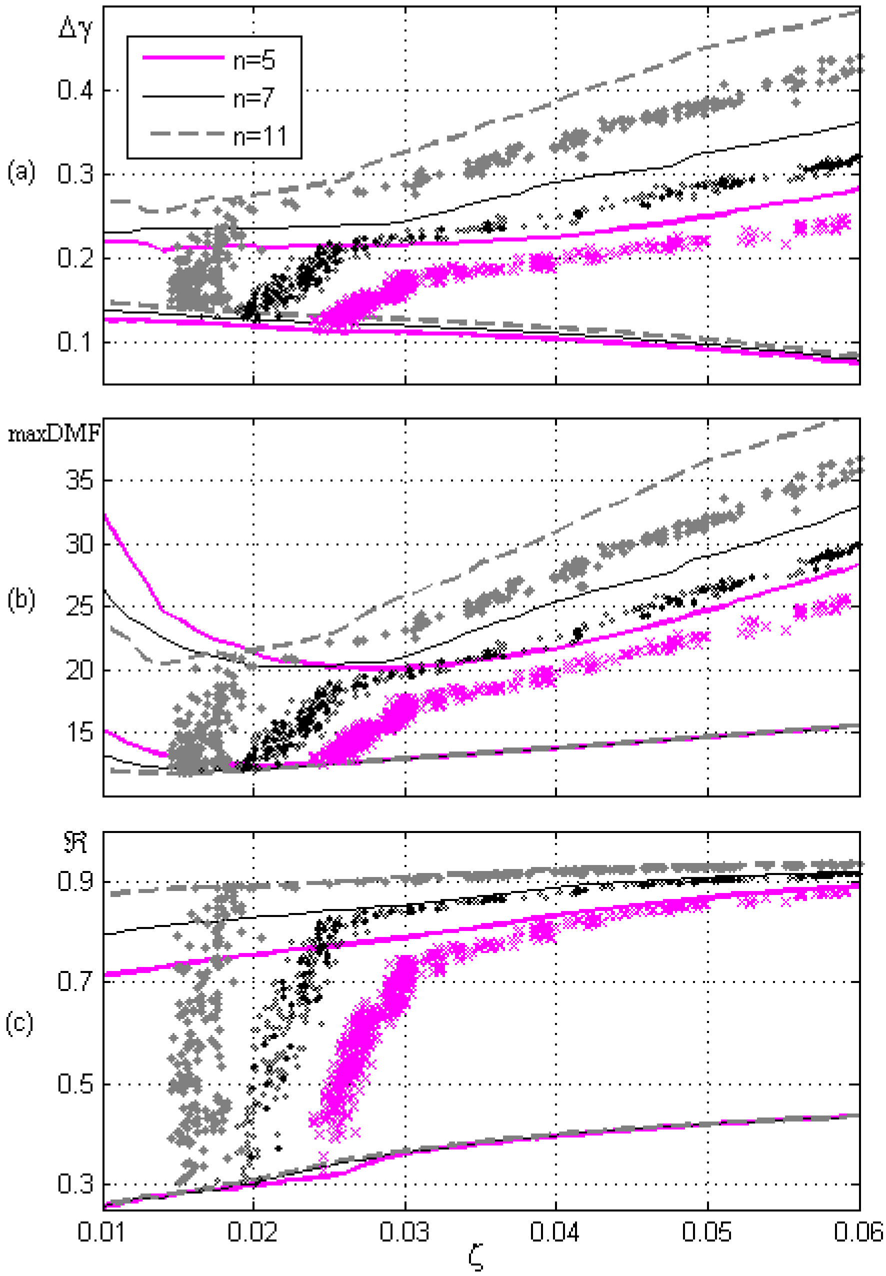

Figure 6. (a) Pareto optimal points in design space within design bounds
(b) Corresponding effectiveness of Pareto optimal solutions within design bounds
(c) Corresponding robustness of Pareto optimal solutions within design bounds

5. COCLUSIONS

In this investigation the optimum design of MTMD systems has been formulated as a multi objective optimization problem. In contrast to most works in which only nominal performance objectives are considered and optimized through numerical search, both effectiveness and robustness have been addressed in this study and optimum sets of solutions have been pursuit through GA optimization. The results of multi-objective GA optimization have been presented using the Pareto optimality concept. These sets of Pareto solutions have been shown to be in perfect accordance with the compromise methodology of MTMD system design presented by the authors in [10]. Indeed, the results validate and strengthen previous results and confine the design compromise to the more restricted set of non-dominated design solutions. According to the designer's preferences and uncertainty level present in the specific application, the most effective MTMD with required robustness can be chosen from the Pareto optimal set of solutions.

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